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# SUPERSYMMETRIC STANDARD MODELS ON D-BRANES

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## ABSTRACT

Type IIB superstring models with the standard model gauge group on D3-branes and with massless matter associated with open strings joining D3-branes to D3-branes or D3-branes to D7<sub>3</sub>-branes are studied. Models with gauge coupling constant unification at an intermediate scale between about  $10^{10}$  and  $10^{12}$  GeV and consistency with the observed value of  $\sin^2 \theta_W(M_Z)$  are obtained. Extra vector-like states and extra pairs of Higgs doublets play a crucial role.

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Until recently, heterotic string theory has been the framework for model building in string theory. It was possible to construct three-generation models with realistic gauge groups in compactifications of the weakly coupled heterotic string on an orbifold. However, there was some difficulty in finding a natural explanation for the discrepancy between the “observed” unification of gauge coupling constants at  $2 \times 10^{16}$  GeV [1] and the string scale of order  $10^{18}$  GeV where unification of coupling constants should occur at tree level [2]. These two distinct scales have been reconciled either by employing the moduli-dependent loop corrections to the gauge kinetic function with a value of the  $T$ -modulus an order of magnitude larger than that obtained from a straightforward minimization of the effective potential (in hidden sector gaugino condensate models) [3], or by introducing extra matter states at an intermediate scale [4], in addition to those of the minimal supersymmetric standard model (MSSM). In the strongly coupled heterotic string theory in the corner of  $M$ -theory corresponding to 11-dimensional supergravity at low energies, this problem is resolved differently through the existence of an extra dimension which allows the string scale to differ substantially from the four-dimensional Planck scale [5].

An alternative way forward, which has been the subject of recent interest, is to employ type IIB superstring theory compactified on an orientifold or orbifold with D-branes present [6, 7]. In that case, the string scale can be altered by adjusting the “radii” of the underlying torus to obtain string scales which differ from the Planck scale. In this context, it is quite natural for the lack of supersymmetry in an anti-D-brane [8] hidden sector to break supersymmetry in the observable sector by gravitational interactions between the two sectors. Since we expect the lack of supersymmetry in the anti-D-brane sector to be characterized by the string scale  $M_s$ , the mass of sparticles in the observable sector will be of order  $M_s^2/M_p$ , where  $M_p$  is the four-dimensional Planck mass. For sparticle masses of order 1 TeV we must have  $M_s$  around  $10^{10}$  to  $10^{12}$  GeV. Then, unification of gauge coupling constants at a scale of this order [9] is to be expected (apart from some subtleties to do with Kaluza-Klein modes and winding modes which we shall touch on later.) Interestingly, it is possible for such models to contain the extra matter required to allow the renormalization group equations to run to unification at this lower scale.

A recent approach to type IIB D-brane model building (the so called bottom-up approach) [10] has been to set up the observable sector gauge group and matter fields on a set of D-branes at a  $R^6/\mathbb{Z}_N$  orbifold singularity *before* embedding this local theory in a global orbifold (or orientifold or Calabi-Yau) theory. The reason why this is an efficient approach to model building is that some properties of the model, such as the number of generations, depend only on the local theory. In this approach, it has been proved possible to obtain three-generation models consistent with the observed value of  $\sin^2 \theta_W$  with D3-branes and D7-branes located at a  $R^6/\mathbb{Z}_3$  singularity and with the local theory embedded in a  $\mathbb{Z}_3$  orbifold, provided that the observable sector gauge group is the left-right symmetric  $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)$ . In the examples studied with standard model gauge group  $SU(3) \times SU(2) \times U(1)$  it did not prove possible to obtain consistency with the experimentally measured  $\sin^2 \theta_W$ . It is our purpose here to construct alternative  $\mathbb{Z}_3$  orbifold compactified type IIB D-brane

models which are consistent with the observed  $\sin^2 \theta_W$  when the gauge group is that of the standard model.

As in [10], in the models we shall study here the gauge fields of the standard model are associated with a set of D3-branes at a  $R^6/\mathbb{Z}_3$  singularity, which we take to be the origin. The action of  $\mathbb{Z}_3$  on complex scalars is given by

$$\theta = \text{diag}(e^{2\pi i b_1/3}, e^{2\pi i b_2/3}, e^{2\pi i b_3/3}) \quad (1)$$

with

$$b_1 = b_2 = -1, b_3 = 2, \quad (2)$$

which respects supersymmetry. The action of  $\theta$  on the Chan-Paton indices of open strings ending on the D3-branes is given by

$$\gamma_{\theta,3} = \text{diag}(I_{n_0}, \alpha I_{n_1}, \alpha^2 I_{n_2}) \quad (3)$$

where the  $n_i$  are non-negative integers and

$$\alpha = e^{2\pi i/3} \quad (4)$$

$\mathbb{Z}_3$  projections can then be made on the gauge field and matter field states. The projection on the gauge bosons gives the gauge group  $U(n_0) \times U(n_1) \times U(n_2)$  and with the choice

$$n_0 = 3, n_1 = 2, n_2 = 1 \quad (5)$$

we *have* the standard model gauge group (up to some  $U(1)$  factors)  $U(3) \times U(2) \times U(1)$ . The projection on the massless matter states gives supermultiplets  $3[(n_0, \bar{n}_1) + (n_1, \bar{n}_2) + (n_2, \bar{n}_0)]$  which with the values (5) of the  $n_i$  is

$$3[(3, 2)_{1/6} + (1, 2)_{1/2} + (\bar{3}, 1)_{-2/3}] \quad (6)$$

where the weak hypercharge has been identified as

$$Y = -\sum_{i=0}^2 \frac{Q_{n_i}}{n_i} = -(\frac{1}{3}Q_3 + \frac{1}{2}Q_2 + Q_1) \quad (7)$$

The model as it stands does not satisfy the twisted tadpole cancellation conditions for a  $\mathbb{Z}_3$  singularity

$$3\text{Tr}\gamma_{\theta,3} - \text{Tr}\gamma_{\theta,7_1} - \text{Tr}\gamma_{\theta,7_2} + \text{Tr}\gamma_{\theta,7_3} = 0 \quad (8)$$

where the  $\gamma_{\theta,7_i}$  are for the D7<sub>i</sub>-branes which overlap the origin, where the D3-branes are located. It is therefore necessary to introduce some D7-branes to achieve the cancellation. We choose to introduce only D7<sub>3</sub>-branes at complex coordinate  $y_3 = 0$ . The general action of  $\theta$  on the Chan-Paton indices of the open strings ending on the D7<sub>3</sub>-branes is given by

$$\gamma_{\theta,7_3} = \text{diag}(I_{u_0^3}, \alpha I_{u_1^3}, \alpha^2 I_{u_2^3}) \quad (9)$$

where the  $u_i^3$  are non-negative integers. Then (8) becomes

$$(9 + u_0^3) + (6 + u_1^3)\alpha + (3 + u_2^3)\alpha^2 = 0 \quad (10)$$

whose general solution is

$$u_0^3 = u, u_1^3 = 3 + u, u_2^3 = 6 + u \quad (11)$$

where  $u$  is a non-negative integer, so that

$$\gamma_{\theta, 7_3} = \text{diag}(I_u, \alpha I_{3+u}, \alpha^2 I_{6+u}) \quad (12)$$

The gauge group associated with the D7<sub>3</sub>-branes is  $U(u) \times U(3+u) \times U(6+u)$  and the associated massless states are in supermultiplets in representations of the gauge group

$$3[(u, \overline{3+u}) + (3+u, \overline{6+u}) + (6+u, \bar{u})] \quad (13)$$

The  $37_3 + 7_3 3$  sector states, arising from open strings with one end on a D3-brane and the other end on a D7<sub>3</sub>-brane, are in non-trivial representations of both the D3-brane and the D7<sub>3</sub>-brane gauge groups and are

$$\begin{aligned} & (3, \overline{3+u})_{-1/3} + (2, \overline{6+u})_{-1/2} + (1, \bar{u})_{-1} \\ & + (u, \bar{2})_{1/2} + (3+u, \bar{1})_1 + (6+u, \bar{3})_{1/3} \end{aligned} \quad (14)$$

(If  $u = 0$  the representations  $(1, \bar{u})_{-1}$  and  $(u, \bar{2})_{1/2}$  are absent.) Combining the  $33$  and  $37_3 + 7_3 3$  matter, the content in terms of representations of  $SU(3) \times SU(2) \times U(1)$  is

$$\begin{aligned} & 3(Q_L + u_L^c + d_L^c + L + H_1 + H_2 + e_L^c) \\ & + (3+u)(d_L^c + \bar{d}_L^c) + u(H_1 + H_2) + u(e_L^c + \bar{e}_L^c) \end{aligned} \quad (15)$$

which is 3 generations plus some extra pairs of Higgs doublets and vector-like quark and lepton matter.

In order to reduce the size of the D7<sub>3</sub>-brane gauge group we specialize to

$$u = 3\tilde{u} \quad (16)$$

where  $\tilde{u}$  is a non-negative integer, and introduce a Wilson line in the second complex plane, embedded in the D7<sub>3</sub>-brane gauge group,

$$\gamma_{W, 7_3} = \text{diag}(I_{\tilde{u}}, \alpha I_{\tilde{u}}, \alpha^2 I_{\tilde{u}}, I_{1+\tilde{u}}, \alpha I_{1+\tilde{u}}, \alpha^2 I_{1+\tilde{u}}, I_{2+\tilde{u}}, \alpha I_{2+\tilde{u}}, \alpha^2 I_{2+\tilde{u}}) \quad (17)$$

This Wilson line may be represented by the shift

$$W_{7_3} = -\frac{1}{3} \left( 0^{\tilde{u}}, 1^{\tilde{u}}, 2^{\tilde{u}}, 0^{1+\tilde{u}}, 1^{1+\tilde{u}}, 2^{1+\tilde{u}}, 0^{2+\tilde{u}}, 1^{2+\tilde{u}}, 2^{2+\tilde{u}} \right) \quad (18)$$

There is then an additional projection on the root vectors  $\rho_a$  for  $7_3 7_3$  sector gauge fields and massless matter states

$$\rho_a \cdot W_{7_3} = 0 \pmod{\mathbb{Z}} \quad (19)$$

This breaks the D7<sub>3</sub>-brane gauge group to  $[U(\tilde{u}) \times U(1 + \tilde{u}) \times U(2 + \tilde{u})]^3$  and the  $7_3 7_3$  sector matter states reduce to

$$3[(\tilde{u}, \overline{1 + \tilde{u}}) + (1 + \tilde{u})(\overline{2 + \tilde{u}}) + (2 + \tilde{u}, \tilde{\tilde{u}})] \quad (20)$$

for *each* of the  $U(\tilde{u}) \times U(1 + \tilde{u}) \times U(2 + \tilde{u})$  factors of the gauge group. The Wilson line does not delete any  $37_3 + 7_3 3$  states because the D3-branes are at the origin and massless  $37_3 + 7_3 3$  states have the ends of the string at the origin. Thus, the influence of the Wilson line is not felt. Decomposing with respect to the  $[U(\tilde{u}) \times U(1 + \tilde{u}) \times U(2 + \tilde{u})]^3$  gauge group, the  $37_3 + 7_3 3$  states are

$$\begin{aligned} & (3, \overline{1 + \tilde{u}})_{-1/3} + (2, \overline{2 + \tilde{u}})_{-1/2} + (1, \tilde{\tilde{u}})_{-1} \\ & + (\tilde{u}, \overline{2})_{1/2} + (1 + \tilde{u}, \overline{1})_1 + (2 + \tilde{u}, \overline{3})_{1/3} \end{aligned} \quad (21)$$

for *each* of the  $U(\tilde{u}) \times U(1 + \tilde{u}) \times U(2 + \tilde{u})$  factors of the gauge group. Yukawa couplings of the  $37_3 + 7_3 3$  states to  $7_3 7_3$  states arise from the superpotential terms

$$\begin{aligned} & \phi_{(2+\tilde{u}, \tilde{\tilde{u}})}^{7_3 7_3} \phi_{(2, 2+\tilde{u})}^{37_3} \phi_{(\tilde{u}, \overline{2})}^{7_3 3} \\ & + \phi_{(\tilde{u}, 1+\tilde{u})}^{7_3 7_3} \phi_{(1, \tilde{\tilde{u}})}^{37_3} \phi_{(1+\tilde{u}, \overline{1})}^{7_3 3} \\ & + \phi_{(1+\tilde{u}, 2+\tilde{u})}^{7_3 7_3} \phi_{(3, 1+\tilde{u})}^{37_3} \phi_{(2+\tilde{u}, \overline{3})}^{7_3 3} \end{aligned} \quad (22)$$

for *each* of the  $U(\tilde{u}) \times U(1 + \tilde{u}) \times U(2 + \tilde{u})$  factors of the gauge group. As can be seen from (20), there are three distinct  $7_3 7_3$  sector multiplets for any given gauge group representations. Mass can be given to some (or all) of the extra vector-like matter states by turning on an expectation value for  $7_3 7_3$  sector scalars. We shall return to the implications for  $\sin^2 \theta_W$  shortly.

To complete the model we embed the  $\mathbb{Z}_3$  singularity at the origin in a  $\mathbb{Z}_3$  orbifold. Because the D7<sub>3</sub>-branes at  $y_3 = 0$  not only overlap the fixed point at the origin but also the fixed points at  $(\pm 1, 0, 0)$ ,  $(0, \pm 1, 0)$  and  $(\pm 1, \pm 1, 0)$ , we must ensure that the twisted tadpole cancellation conditions are also satisfied at these fixed points. In the presence of the Wilson line in the second complex plane the twisted tadpole conditions are modified to

$$3\text{Tr}\gamma_{\theta,3} + \text{Tr}(\gamma_{\theta,7_3} \gamma_{W,7_3}^p) = 0 \quad (23)$$

(assuming only D3-branes and D7<sub>3</sub>-branes) where  $p = 0$  for  $(\pm 1, 0, 0)$  and  $p = \pm 1$  for  $(0, \pm 1, 0)$  and  $(\pm 1, \pm 1, 0)$ . However,

$$\text{Tr}(\gamma_{\theta,7_3} \gamma_{W,7_3}^p) = 0 \quad (24)$$

for  $p = \pm 1$ . Thus, the twisted tadpole conditions are already satisfied at  $(0, \pm 1, 0)$  and  $(\pm 1, \pm 1, 0)$ . We can arrange for the twisted tadpole conditions to be satisfied at

$(\pm 1, 0, 0)$  by including D3-branes at these fixed points. One suitable choice is to add 6 D3-branes with  $\gamma_{\theta,3}$  of the same form

$$\gamma_{\theta,3} = \text{diag}(I_3, \alpha I_2, \alpha^2 I_1) \quad (25)$$

as at the origin at each of the fixed points  $(\pm 1, 0, 0)$ . Another simple choice would be to add 3 D3-branes at each of the fixed points with

$$\gamma_{\theta,3} = \text{diag}(I_2, \alpha I_1) \quad (26)$$

For definiteness we considered the choice (25). There is then a total of 18 D3-branes and a total of  $9 + 9\tilde{u}$  D7<sub>3</sub>-branes. Finally, to cancel untwisted tadpoles, it is necessary to add an equal number of  $\overline{\text{D3}}$ -branes to balance the D3-branes and an equal number of  $\overline{\text{D7}}_3$ -branes to balance the D7<sub>3</sub>-branes. We would also like to avoid overlap of the anti-branes with the observable sector D3-branes and D7<sub>3</sub>-branes at the origin and overlapping the origin, respectively. This is easily achieved by placing the  $\overline{\text{D3}}$ -branes and  $\overline{\text{D7}}_3$ -branes at complex coordinate  $y_3 \neq 0$ . For example, we might place 18  $\overline{\text{D3}}$ -branes with

$$\gamma_{\theta,\bar{3}} = \text{diag}(I_6, \alpha I_6, \alpha^2 I_6) \quad (27)$$

at a single fixed point with  $y_3 \neq 0$ , and  $9 + 9\tilde{u}$   $\overline{\text{D7}}_3$ -branes at  $y_3 = 1$  or  $(y_3 = -1)$  with

$$\gamma_{\theta,\bar{7}_3} = \text{diag}(I_{3+3\tilde{u}}, \alpha I_{3+3\tilde{u}}, \alpha^2 I_{3+3\tilde{u}}) \quad (28)$$

Because  $\text{Tr}\gamma_{\theta,\bar{3}}$  and  $\text{Tr}\gamma_{\theta,\bar{7}_3}$  are both zero, the twisted tadpole conditions continue to be satisfied. As a consequence of the geometrical separation of brane and anti-brane sectors, the anti-D-brane sectors, in which supersymmetry is absent, communicate only gravitationally with the observable D-brane sectors. As discussed earlier, we then expect unification of gauge couplings at a scale of  $10^{10}$  to  $10^{12}\text{GeV}$ .

We turn next to the consistency of such models with the observed value of  $\sin^2 \theta_W$ . In general, if we allow for running of the observable gauge coupling constants to a unification scale  $M_X$ , and if, for greater generality, we also allow that any “light” extra matter states, over and above those of the MSSM, have masses on a scale  $M_Y$  between  $M_Z$  and  $M_X$ , then, in a supersymmetric theory,

$$\begin{aligned} \sin^2 \theta_W(M_Z) &= \frac{3}{14} \left[ 1 + \frac{11}{6\pi} \alpha(M_Z) (b_2 - \frac{3}{11} b_1) \ln \frac{M_X}{M_Y} \right. \\ &\quad \left. + \frac{11}{6\pi} \alpha(M_Z) (\tilde{b}_2 - \frac{3}{11} \tilde{b}_1) \ln \frac{M_Y}{M_Z} \right] \end{aligned} \quad (29)$$

and

$$\begin{aligned} \alpha_3^{-1}(M_Z) &= \frac{3}{14} \left[ \alpha^{-1}(M_Z) - \frac{1}{2\pi} (b_1 + b_2 - \frac{14}{3} b_3) \ln \frac{M_X}{M_Y} \right. \\ &\quad \left. - \frac{1}{2\pi} (\tilde{b}_1 + \tilde{b}_2 - \frac{14}{3} \tilde{b}_3) \ln \frac{M_Y}{M_Z} \right] \end{aligned} \quad (30)$$

or, equivalently,

$$\begin{aligned}
\sin^2 \theta_W(M_Z) &= \frac{3}{14} + \frac{11}{3} \frac{(b_2 - \frac{3}{11}b_1)}{(b_1 + b_2 - \frac{14}{3}b_3)} \left[ \frac{3}{14} - \alpha(M_Z)\alpha_3^{-1}(M_Z) \right] \\
&+ \frac{3}{14} \frac{11}{6\pi} \frac{\alpha(M_Z)}{(b_1 + b_2 - \frac{14}{3}b_3)} \left[ (b_1 + b_2 - \frac{14}{3}b_3)(\tilde{b}_2 - \frac{3}{11}\tilde{b}_1) \right. \\
&- \left. (b_2 - \frac{3}{11}b_1)(\tilde{b}_1 + \tilde{b}_2 - \frac{14}{3}\tilde{b}_3) \right] \ln\left(\frac{M_Y}{M_Z}\right)
\end{aligned} \tag{31}$$

and

$$\begin{aligned}
\frac{3}{14} \frac{\alpha(M_Z)}{2\pi} \ln \frac{M_X}{M_Y} &= (b_1 + b_2 - \frac{14}{3}b_3)^{-1} \left[ \frac{3}{14} - \alpha(M_Z)\alpha_3^{-1}(M_Z) \right] \\
&- \frac{3}{14} \frac{\alpha(M_Z)}{2\pi} \frac{(\tilde{b}_1 + \tilde{b}_2 - \frac{14}{3}\tilde{b}_3)}{(b_1 + b_2 - \frac{14}{3}b_3)} \ln \frac{M_Y}{M_Z}
\end{aligned} \tag{32}$$

In these equations,  $\frac{3}{14}$  is the value of the  $\sin^2 \theta_W$  at the unification scale in models where the D3-branes that carry the gauge group of the standard model are at a  $\mathbb{Z}_3$  singularity. The renormalization group coefficients for  $SU(3) \times SU(2) \times U(1)$  are given in terms of Casimirs for the group factors  $G_a$  and its massless matter field representations  $R_a$  by

$$b_a = -3C_1(G_a) + \sum_{R_a} C_2(R_a), \quad a = 1, 2, 3 \tag{33}$$

The renormalization group coefficients for scales between  $M_Z$  and  $M_Y$ , which we denote by  $\tilde{b}_a$ , are those of the MSSM.

So far as the observable sector is concerned, the massless matter content is as in (15) with  $u = 3\tilde{u}$ . Let us first consider the case where we do not turn on any expectation values for the  $7_3 7_3$  sector scalars, so that all of the extra matter in (15), over and above that of MSSM, contributes to the running of the gauge coupling constants. If we do not introduce the extra scale  $M_Y$  ( $M_Y = M_Z$ ), then

$$b_3 = 3\tilde{u}, \quad b_2 = 3(\tilde{u} + 1), \quad b_1 = 15 + 11\tilde{u} \tag{34}$$

In that case, because  $b_2 - \frac{3}{11}b_1$  is negative,  $\sin^2 \theta_W(M_Z)$  is less than  $\frac{3}{14} = 0.214$ , whereas the observed value is 0.231. The correction due to the running of the gauge coupling constants is in the wrong direction. For the renormalization group coefficients of the MSSM,  $\tilde{b}_2 - \frac{3}{11}\tilde{b}_1$  is also negative so that including the extra scale  $M_Y$  does not help.

Suppose next that the  $7_3 7_3$  sector scalars give mass to  $\alpha$  copies of  $e_L^c + \bar{e}_L^c$ ,  $\beta$  copies of  $H_1 + H_2$ , and  $\gamma$  copies of  $d_L^c + \bar{d}_L^c$  on a scale larger than  $M_X$ . This requires  $\alpha \leq 3\tilde{u}$ ,  $\beta < 3(\tilde{u} + 1)$  and  $\gamma \leq 3(\tilde{u} + 1)$ . We use the values [11]  $\alpha^{-1}(M_Z) = 128.9$  and  $\alpha_3(M_Z) = 0.119$ . Then, if we do not introduce an extra mass scale  $M_Y$  (i.e.  $M_Y = M_Z$ ), the best value for  $\sin^2 \theta_W(M_Z)$  is 0.2275, which occurs when  $\alpha - \beta = 2$  and  $\gamma - \beta = 4$ . Moreover, the unification scale  $M_X \simeq 1.3 \times 10^{10} \text{GeV}$ , which is consistent

with observed supersymmetry breaking arising from gravitational interactions with an anti-brane sector. To avoid  $\alpha_3$  becoming infinite at a lower scale it is necessary to choose  $\tilde{u} \leq 2$ , but the above constraints on  $\alpha, \beta$  and  $\gamma$  can easily be satisfied.

Finally, let us suppose instead that expectation values of some  $7_3 7_3$  sector scalars give mass to three copies of  $d_L^c + \bar{d}_L^c$  and three copies of  $e_L^c + \bar{e}_L^c$  on a scale larger than  $M_X$ . Let us also suppose that some other  $7_3 7_3$  sector scalars give mass to the extra matter, over and above that of the MSSM, on a scale  $M_Y$  not very much larger than  $M_Z$ . Then we obtain the observed value of  $\sin^2 \theta_W(M_Z) = 0.231$  with

$$\frac{M_Y}{M_Z} = 13.9 \quad (35)$$

and

$$M_X \sim 1.1 \times 10^{12} \text{ GeV} \quad (36)$$

This is consistent with supersymmetry breaking being transmitted gravitationally from an anti-brane sector provided the string scale is the unification scale. Assuming that the compactification is isotropic with a compactification scale  $M_c$ , then the string scale  $M_s$  is given by [12]

$$\frac{M_s^4}{M_c^3} = \alpha_{D3} \frac{M_p}{\sqrt{2}} \quad (37)$$

where  $\alpha_{D3}$  is the value of  $\frac{g^2}{4\pi}$  at unification when the observable gauge group is on D3-branes and  $M_p$  is the Planck mass. Winding modes have mass  $M_w$  given by

$$M_w = \frac{M_s^2}{M_c} \quad (38)$$

To estimate the value of  $\alpha_{D3}$ , we run the QCD fine structure constant  $\alpha_3$  from  $M_Z$  to  $M_X \approx 1.1 \times 10^{12} \text{ GeV}$ . To avoid  $\alpha_3$  becoming infinite at a lower scale it is necessary to choose  $\tilde{u} \leq 1$ . This restricts us to

$$\tilde{u} = 1 \quad (39)$$

because  $\tilde{u} = 0$  does not allow  $7_3 7_3$  sector expectation values to give mass to any copies of  $e_L^c + \bar{e}_L^c$ . This can be seen from (22). With the  $\tilde{u} = 1$  case,  $\alpha_3$  runs only between  $M_Z$  and  $M_Y$  and its value at  $M_X$  is approximately the same as at  $M_Z$ . Taking  $M_s \approx 1.1 \times 10^{12} \text{ GeV}$ , then  $M_w \sim 10^2 M_s$  and the scale associated with winding modes is above the string scale. The compactification scale  $M_c$  is not directly relevant to unification because there are no Kaluza-Klein modes when all the boundary conditions for the compact manifold are Dirichlet. Thus, we may take the string scale to be the unification scale.

The situation with regard to mass hierarchies for the quark and lepton masses is the same as for the model discussed in [10]. Lepton mass terms require the coupling in the superpotential of the chiral fields  $L, e_L^c$  and  $H_1$  (possibly accompanied by some



$7_3 7_3$  sector fields and/or  $3' 7_3 + 7_3 3'$  fields, which are uncharged with respect to the standard model gauge group; ( $3'$  refers to non-standard-model D3-branes.) Such couplings are allowed (as usual) by conservation of weak hypercharge, but they are *not* allowed by conservation of the other two  $U(1)$  charges originating from the standard model gauge group. It is expected that the global conservation of all  $U(1)$  charges, including those originating from the  $D7_3$ -brane and  $D3'$ -brane gauge groups, will survive after the anomalous  $U(1)$  gauge symmetries have been broken by a modified Green-Schwarz mechanism. For the lepton mass terms above it is obvious that  $Q_2$  is not conserved, and the inclusion of other standard model gauge singlet fields does not alleviate the problem. However, the global symmetries do allow the coupling

$$Le_L^c \hat{H}_1 = \phi_{(2, \bar{u}+2)}^{37_3} \phi_{(\bar{u}+2, \bar{u})}^{7_3 7_3} \phi_{(\bar{u}, \bar{u}+1)}^{7_3 7_3} \phi_{(\bar{u}+1, \bar{1})}^{7_3 3} \phi_{(1, \bar{u})}^{37_3} \phi_{(\bar{u}, 2)}^{7_3 3} M_s^{-3} \quad (40)$$

where  $\hat{H}_1$  is an effective Higgs field

$$\hat{H}_1 = \phi_{(\bar{u}+2, \bar{u})}^{7_3 7_3} \phi_{(\bar{u}, \bar{u}+1)}^{7_3 7_3} \phi_{(1, \bar{u})}^{37_3} \phi_{(\bar{u}, 2)}^{7_3 3} M_s^{-3} \quad (41)$$

A similar discussion shows that baryon number  $B$  is (perturbatively) conserved in this model since  $B = \frac{1}{3}Q_3$ . Thus the proton is absolutely stable, and this is also the case in the model of reference [10]. On the other hand these global  $U(1)$  symmetries do *not* forbid the lepton-number non-conserving, dimension 4 operator  $Q_L d_L^c L$ , nor some lepton-number non-conserving, dimension 6 operators. Some other resolution of the problem will have to be found. In this respect the present model cannot improve upon that of reference [10].

It is interesting to note that the models discussed here possess dual models in which the observable sector gauge group is on  $D7_3$ -branes at  $y_3 = 0$  instead of  $D3$ -branes at the origin. So far as the observable sector matter is concerned  $D3$ -branes and  $D7_3$ -branes are replaced by  $D7_3$ -branes and  $D7_1$ -branes. (Strictly, we should recast the models we have been discussing in terms of  $D3$ -branes and  $D7_2$ -branes to obtain the duality.) The dual models have action on the Chan-Paton indices

$$\gamma_{\theta, 7_3} = \text{diag}(I_3, \alpha I_2, \alpha^2 I_1) \quad (42)$$

for  $D7_3$ -branes at  $y_3 = 0$  and

$$\gamma_{\theta, 7_1} = -\text{diag}(\alpha^2 I_{2+\bar{u}}, I_{\bar{u}}, \alpha I_{\bar{u}+1}) \quad (43)$$

for  $D7_1$ -branes at  $y_1 = 0, \pm 1$ . Notice that  $9 + 9\tilde{u}$   $D7_3$ -branes all at  $y_3 = 0$  with the addition of a Wilson line, in the original version, is replaced by  $3 + 3\tilde{u}$   $D7_1$ -branes at each of  $y_1 = 0, \pm 1$  with no Wilson line, in the dual version. As before, the embedding in an orbifold is more model dependent.

In conclusion, we have constructed type IIB D-brane models with the standard model gauge group on  $D3$ -branes and the massless matter states for the standard model in  $33$  and  $37_3 + 7_3 3$  sectors. These models allow unification of gauge coupling constants at an intermediate scale of order  $10^{10}$  to  $10^{12}$  GeV, consistently with the observed value of  $\sin^2 \theta_W(M_Z)$ . Additional vector-like states and pairs of Higgs doublets with low mass compared to the unification scale play a crucial role.

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